On Brune's Tests

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***Abstract*—Brune's tests are well known in circuit theory. They are two simple voltage tests used to ascertain whether the overall component stemming from the parallel–parallel, series–series, series–parallel, or parallel–series connection of two four-terminal components has a two-port matrix that is equal to the sum of the appropriate individual two-port matrices. What is less known is that Brune's tests only are necessary and sufficient for reciprocal components and that all the extensions that are available in literature to also cover the nonreciprocal components are inaccurate. In this brief, a rigorous and thorough unified analysis of the four connections is carried out. It provides the complete characterization of the overall component under a two-port operation in the general case and, most importantly, the exact parameter conditions and voltage/current tests for the validity of the matrix sum rule.**

***Index Terms*—Brune's tests, circuit theory, four-terminal components, two-port operation.**

I. Introduction

In 1929, Strecker and Feldtkeller [1] pioneered the six two-port matrix representations of four-terminal components. They also derived the classical formulas for the two-port matrix of component Q stemming from the parallel–parallel and series–series connections of two four-terminal components A and B (see Fig. 1), i.e., $\boldsymbol{Y}_Q = \boldsymbol{Y}_A + \boldsymbol{Y}_B$ and $\boldsymbol{Z}_Q = \boldsymbol{Z}_A + \boldsymbol{Z}_B$. They warned that these formulas fail if the circuit schemes of A and B are inadequate (e.g., if some inner components of A or B short-circuit because of the connection).

Two years later, Selach [2] presented several examples of the four connections of two four-terminal components for which the matrix sum rule did or did not apply, but he gave no general validity criterion.

In 1932, Baerwald [3] argued that the reason for the failures observed was that, after the connection, the two four-terminal components no longer worked as two-ports since a circulating current disturbed the ports. In his rather involved analysis, he adopted, as a scheme of each four-terminal component, a "complete square" of six admittances and derived two conditions for the matrix sum rule to apply in the parallel–parallel and series–series connections. According to the hypotheses, these results are correct for the reciprocal components only, but Baerwald claimed that they were also correct for the nonreciprocal components.

Soon after, Brune [4] suggested that the circulating current was a direct consequence of the imbalance between the "third voltages" of the two four-terminal components under individual two-port operation. Thus, for the parallel–parallel, series–series, and series–parallel connections, he derived a simple criterion, which is expressed by two equalities between two parameters that were purposely chosen for each connection. He also provided a circuit interpretation of these parameter conditions, but he did it in words and without schemes. Note that Brune did not even mention reciprocity. In his reply, Baerwald [5] recognized the greater simplicity of Brune's tests over his own, but he emphasized the inconvenience of changing the representation of the two four-terminal components in accordance with the connection concerned. This opinion found no followers, as the widespread success of Brune's tests has confirmed.

In 1935, Guillemin [6] mimicked Brune's analysis, and he was probably the first who presented the two circuit schemes and voltage tests for the four connections that are currently associated with Brune's name. Since then, these schemes and tests have been included in all high-level textbooks on circuit theory, e.g., see [7] and [10].

In 1976, Horrocks and Nightingale [8], using an analytical approach, reconsidered the parallel–parallel connection in the nonreciprocal case. They obtained the old Brune's pair of parameter conditions and a new pair. They claimed that both pairs were sufficient but not necessary in general, whereas they were necessary and sufficient (and coincident) in the reciprocal case. The latter conclusion is true, and the former is not.

In 1978, Buijze [9] showed by means of two simple examples that, in the nonreciprocal case, Brune's tests are neither necessary nor sufficient. He made a direct analysis of the series–series connection and obtained the correct equation set. Unfortunately, he failed in discussing the solutions of this equation set; thus, the pair of parameter conditions he gave as necessary and sufficient in fact are not. Note that Buijze's pair is equivalent to Horrocks and Nightingale's new pair.

In this brief, a rigorous and thorough analysis of the four connections is performed, which provides in the end the true voltage/current tests for the matrix sum rule to apply. It rests on a unified approach that takes into consideration the most general representation of the two four-terminal components.

This brief is organized as follows. First, the assumptions on components A and B are stated, and the consequences on their representations are derived (see Section II). Second, the complete characterization of Q_{TP}, i.e., component Q under a two-port operation, is provided (see Section III), and the parameter conditions are deduced (see Section IV). Third, the test circuits are analyzed (see Section V) and the voltage/current tests are presented (see Section VI). Finally, some conclusions are drawn (see Section VII).

II. Assumptions and Representations

Two basic assumptions are adopted in this brief. First, either four-terminal component is required to be linear algebraic, i.e.,

Manuscript received August 29, 2013; accepted January 18, 2014. Date of publication February 28, 2014; date of current version April 16, 2014. This brief was recommended by Associate Editor N. Krishnapura.

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Digital Object Identifier 10.1109/TCSII.2014.2304876

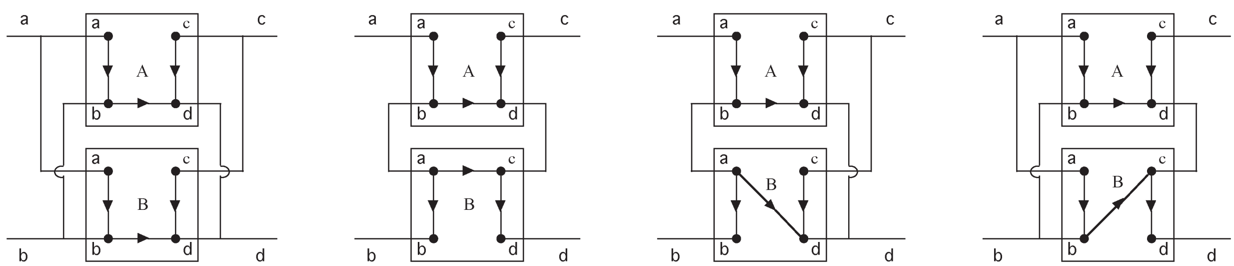

Fig. 1. Parallel–parallel, series–series, series–parallel, and parallel–series connections. The graphs shown are the graphs used in the analysis.

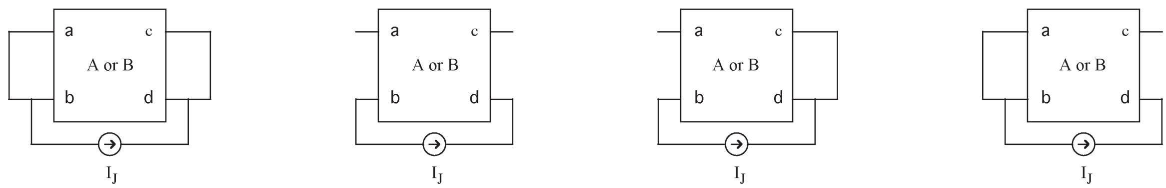

Fig. 2. Circuits (C_{0A}/C_{0B}) used to ascertain whether the two four-terminal components A and B are under-controlled, duly controlled, or over-controlled as far as the parallel–parallel, series–series, series–parallel, and parallel–series connections are concerned.

to obey the following implicit full representation

$$\boldsymbol{M}\begin{bmatrix}\boldsymbol{u}\\ V_{xy}\\ \boldsymbol{w}\\ I_{xy}\end{bmatrix}=\boldsymbol{0}. \tag{1}$$

Here, $\boldsymbol{M}$ is a 6×6 constant matrix; $\boldsymbol{w}$ and $\boldsymbol{u}$ are the two-port input and output variable vectors pertaining to the connection of concern, respectively; x and y are two terminals belonging to different ports (i.e., $x \in \{a, b\}$ and $y \in \{c, d\}$); V_{xy} is the voltage across terminals x and y; and I_{xy} is the current associated with it. For example, in the parallel–parallel connection, for both components, $\boldsymbol{w} = \boldsymbol{v} := [V_{ab}\ V_{cd}]'$, $\boldsymbol{u} = \boldsymbol{i} := [I_{ab}\ I_{cd}]'$, $x = b$, and $y = d$, and then, $V_{xy} = V_{bd}$, and $I_{xy} = I_{bd}$ (primed vectors mean transposed vectors).

Second, either four-terminal component, under a two-port operation ($I_{xy} = 0$), is required to have the two-port representation suited to the connection of concern, i.e.,

$$\boldsymbol{u} = \boldsymbol{K}\boldsymbol{w} \tag{2}$$

where $\boldsymbol{K}$ is a 2×2 constant matrix. For example, in the parallel–parallel connection, both components must have the voltage-controlled two-port representation; hence, for both $\boldsymbol{K} = \boldsymbol{Y}$.

The question arises as to which explicit full representations components A and B can exhibit under assumptions (1) and (2). It is not difficult to realize that, in order for these assumptions to be consistent, the rank of matrix $\boldsymbol{M}$ must equal 2, 3, or 4. The corresponding explicit full representations of the components will be

$$\boldsymbol{u} = [\boldsymbol{K}\quad \boldsymbol{c}\quad 0]\begin{bmatrix}\boldsymbol{w}\\ I_{xy}\\ V_{xy}\end{bmatrix} \tag{3}$$

$$\begin{bmatrix}\boldsymbol{u}\\ V_{xy}\end{bmatrix}=\begin{bmatrix}\boldsymbol{K} & \boldsymbol{c}\\ \boldsymbol{r} & k\end{bmatrix}\begin{bmatrix}\boldsymbol{w}\\ I_{xy}\end{bmatrix} \tag{4}$$

$$\begin{bmatrix}\boldsymbol{u}\\ V_{xy}\\ I_{xy}\end{bmatrix}=\begin{bmatrix}\boldsymbol{K}\\ \boldsymbol{r}\\ \boldsymbol{0}\end{bmatrix}\boldsymbol{w} \tag{5}$$

where $\boldsymbol{c}$, $\boldsymbol{r}$, and k are a column vector, a row vector, and a scalar, respectively. These representations and the corresponding components will be called under-controlled, duly controlled, and over-controlled, respectively. Notice that reciprocity can only occur in the duly controlled and over-controlled representations.

The check on the rank of the $\boldsymbol{M}$ matrix of components A and B can be replaced by tests performed on proper circuits. In fact, in each circuit of the four shown in Fig. 2 and denoted by C_0, the specific condition $\boldsymbol{w} = \boldsymbol{0}$ pertaining to the parallel–parallel, series–series, series–parallel, and parallel-series connections, respectively, is met, but according to (3)–(5) each circuit is nonuniquely solvable, uniquely solvable, or unsolvable, depending on whether the rank of $\boldsymbol{M}$ equals 2, 3, or 4, respectively. Therefore, under (1) and (2), one has the following double implications hold:

$$\text{rank } \boldsymbol{M} = 2 \quad \Leftrightarrow \quad C_0 \text{ is nonuniquely solvable} \tag{6a}$$
$$\text{rank } \boldsymbol{M} = 3 \quad \Leftrightarrow \quad C_0 \text{ is uniquely solvable} \tag{6b}$$
$$\text{rank } \boldsymbol{M} = 4 \quad \Leftrightarrow \quad C_0 \text{ is unsolvable.} \tag{6c}$$

Hence, under assumptions (1) and (2), checking for the solution of circuits C_{0A} and C_{0B} permits one to ascertain whether A and B are under-controlled, duly controlled, or over-controlled.

III. Complete Characterization of Q_{TP}

By referring to Fig. 1, one easily realizes that the terminals x and y of components A and B, and their graphs, are chosen in a way that, under a two-port operation, the four connections share the following topological constraints:

$$\boldsymbol{u}_Q = \boldsymbol{u}_A + \boldsymbol{u}_B \qquad \boldsymbol{w}_Q = \boldsymbol{w}_A = \boldsymbol{w}_B \tag{7a}$$
$$V_{bd_A} = V_{xy_B} \qquad I_{bd_A} + I_{xy_B} = 0. \tag{7b}$$

Then, one has to consider the configurations that occur depending on whether components A and B are under-controlled, duly controlled, or over-controlled. In fact, nine configurations are possible, but only six are distinct. With a self-explanatory notation, they will be denoted by DD, OO, OD, UU, UD, and OU. To the author's knowledge, only the DD configuration has been considered in literature.

To simplify further calculations, also set

$$\boldsymbol{I}_\perp := \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} \tag{8}$$

and note that, for any column vector $\boldsymbol{z}$, $\boldsymbol{z}'\boldsymbol{I}_\perp\boldsymbol{z} = 0$ holds.

A. DD Configuration

From (4) and (7), after some manipulation, one obtains

$$\boldsymbol{u}_Q = (\boldsymbol{K}_A + \boldsymbol{K}_B)\boldsymbol{w}_Q + (\boldsymbol{c}_A - \boldsymbol{c}_B)I_{bd_A} \tag{9a}$$
$$0 = (\boldsymbol{r}_A - \boldsymbol{r}_B)\boldsymbol{w}_Q + (k_A + k_B)I_{bd_A}. \tag{9b}$$

The following three cases arise.

If $k_A + k_B \neq 0$, then replace

$$I_{bd_A} = -\frac{\boldsymbol{r}_A - \boldsymbol{r}_B}{k_A + k_B}\boldsymbol{w}_Q \tag{10}$$

into (9a) and obtain

$$\boldsymbol{u}_Q = \left[\boldsymbol{K}_A + \boldsymbol{K}_B - \frac{(\boldsymbol{r}_A - \boldsymbol{r}_B)\,(\boldsymbol{c}_A - \boldsymbol{c}_B)}{k_A + k_B}\right]\boldsymbol{w}_Q. \tag{11}$$

Hence, Q_{TP} is $\boldsymbol{w}$-controlled, but the matrix sum rule applies if and only if $\boldsymbol{r}_A = \boldsymbol{r}_B$ or $\boldsymbol{c}_A = \boldsymbol{c}_B$ (I_{bd_A} is zero if and only if $\boldsymbol{r}_A = \boldsymbol{r}_B$).

If $k_A + k_B = 0$ and $\boldsymbol{r}_A \neq \boldsymbol{r}_B$, then Q_{TP} is not $\boldsymbol{w}$-controlled as the two components of $\boldsymbol{w}_Q$ are bound to each other. More precisely, if $\boldsymbol{c}_A \neq \boldsymbol{c}_B$, then Q_{TP} is otherwise duly controlled (i.e., it has other two-port representations) as its equations read

$$(\boldsymbol{c}_A - \boldsymbol{c}_B)'\boldsymbol{I}_\perp\boldsymbol{u}_Q = (\boldsymbol{c}_A - \boldsymbol{c}_B)'\boldsymbol{I}_\perp(\boldsymbol{K}_A + \boldsymbol{K}_B)\boldsymbol{w}_Q \tag{12a}$$
$$(\boldsymbol{r}_A - \boldsymbol{r}_B)\boldsymbol{w}_Q = 0 \tag{12b}$$

(I_{bd_A} is nonzero); if $\boldsymbol{c}_A = \boldsymbol{c}_B$, then Q_{TP} is over-controlled (i.e., $\boldsymbol{u}_Q$ and one component of $\boldsymbol{w}_Q$ depend on the other component of $\boldsymbol{w}_Q$) as its equations read

$$\boldsymbol{u}_Q = (\boldsymbol{K}_A + \boldsymbol{K}_B)\boldsymbol{w}_Q \tag{13a}$$
$$(\boldsymbol{r}_A - \boldsymbol{r}_B)\boldsymbol{w}_Q = 0 \tag{13b}$$

(I_{bd_A} is indeterminable).

If $k_A + k_B = 0$ and $\boldsymbol{r}_A = \boldsymbol{r}_B$, then Q_{TP} is $\boldsymbol{w}$-controlled and, concurrently, the matrix sum rule applies if and only if $\boldsymbol{c}_A = \boldsymbol{c}_B$ (I_{bd_A} is indeterminable). Instead, if $\boldsymbol{c}_A \neq \boldsymbol{c}_B$, then Q_{TP} is under-controlled (i.e., one component of $\boldsymbol{u}_Q$ depends on the other component of $\boldsymbol{u}_Q$ and on $\boldsymbol{w}_Q$) as its equation turns out to be (12a) (I_{bd_A} is nonzero).

B. OO and OD Configurations

From (4), (5), and (7), after some manipulation, one obtains (13) (I_{bd_A} is zero). Hence, Q_{TP} is $\boldsymbol{w}$-controlled and, concurrently, the matrix sum rule applies if and only if $\boldsymbol{r}_A = \boldsymbol{r}_B$. Instead, if $\boldsymbol{r}_A \neq \boldsymbol{r}_B$, then Q_{TP} is over-controlled.

C. UU and UD Configurations

From (3), (4), and (7), after some manipulation, one obtains (9a). Hence, Q_{TP} is $\boldsymbol{w}$-controlled and, concurrently, the matrix sum rule applies if and only if $\boldsymbol{c}_A = \boldsymbol{c}_B$ (I_{bd_A} is indeterminable). Instead, if $\boldsymbol{c}_A \neq \boldsymbol{c}_B$, then Q_{TP} is under-controlled as its equation turns out to be (12a) (I_{bd_A} is nonzero).

D. OU Configuration

From (3), (5), and (7), after some manipulation, one obtains (13a) (I_{bd_A} is zero). Hence, Q_{TP} is $\boldsymbol{w}$-controlled and the matrix sum rule applies with no condition on parameters.

IV. Unified Parameter Conditions

According to the outcomes in Section III, the following main result can be stated: Under assumptions (1) and (2), Q_{TP} is $\boldsymbol{w}$-controlled and the matrix sum rule applies if and only if one of the following condition sets holds:

$$\text{DD}: \quad \boldsymbol{r}_A = \boldsymbol{r}_B \quad \boldsymbol{c}_A = \boldsymbol{c}_B \tag{14a}$$
$$\text{DD}: \quad \boldsymbol{r}_A = \boldsymbol{r}_B \quad k_A + k_B \neq 0 \tag{14b}$$
$$\text{DD}: \quad \boldsymbol{c}_A = \boldsymbol{c}_B \quad k_A + k_B \neq 0 \tag{14c}$$
$$\text{OO, OD}: \quad \boldsymbol{r}_A = \boldsymbol{r}_B \tag{14d}$$
$$\text{UU, UD}: \quad \boldsymbol{c}_A = \boldsymbol{c}_B \tag{14e}$$
$$\text{OU}: \quad \text{——}. \tag{14f}$$

From the outcomes in Section III, it also follows that (14a) with $k_A + k_B \neq 0$, (14b), (14d), and (14f) correspond to a zero circulating current; (14c) with $\boldsymbol{r}_A \neq \boldsymbol{r}_B$ corresponds to a nonzero circulating current; and (14a) with $k_A + k_B = 0$ and (14e) correspond to an indeterminable circulating current. Hence, the occurrence of a nonzero or even of an indeterminable circulating current not necessarily invalidates the matrix sum rule. Moreover, the vanishing of the circulating current is not sufficient to make the matrix sum rule apply (consider the OO and OD configurations).

The only consequence of reciprocity on the parameters appearing in the admissible cases [see (14a)–(14d)] is that, in the DD configuration, the following double implication holds:

$$\boldsymbol{r}_A = \boldsymbol{r}_B \quad \Leftrightarrow \quad \boldsymbol{c}_A = \boldsymbol{c}_B. \tag{15}$$

Hence, for the reciprocal components, (14) is reduced to

$$\text{DD, OO, OD}: \quad \boldsymbol{r}_A = \boldsymbol{r}_B. \tag{16}$$

Evidently, (16) corresponds to either a zero or an indeterminable circulating current (the latter case only in the DD configuration with $k_A + k_B = 0$).

Brune's parameter condition pair corresponds to "DD : $\boldsymbol{r}_A = \boldsymbol{r}_B$," which, in fact, directly follows from balancing, under individual two-port operation, the "third voltages" of the two four-terminal components in Brune's configuration. However, as evidenced earlier, the circulating current not necessarily vanishes.

Horrocks and Nightingale's new condition pair and Buijze's condition pair can be shown to correspond to "DD : $\boldsymbol{c}_A = \boldsymbol{c}_B$."

V. Test Circuits

For each connection, five single input single output circuits can be devised, whose output variables (V_m or I_m, as to the

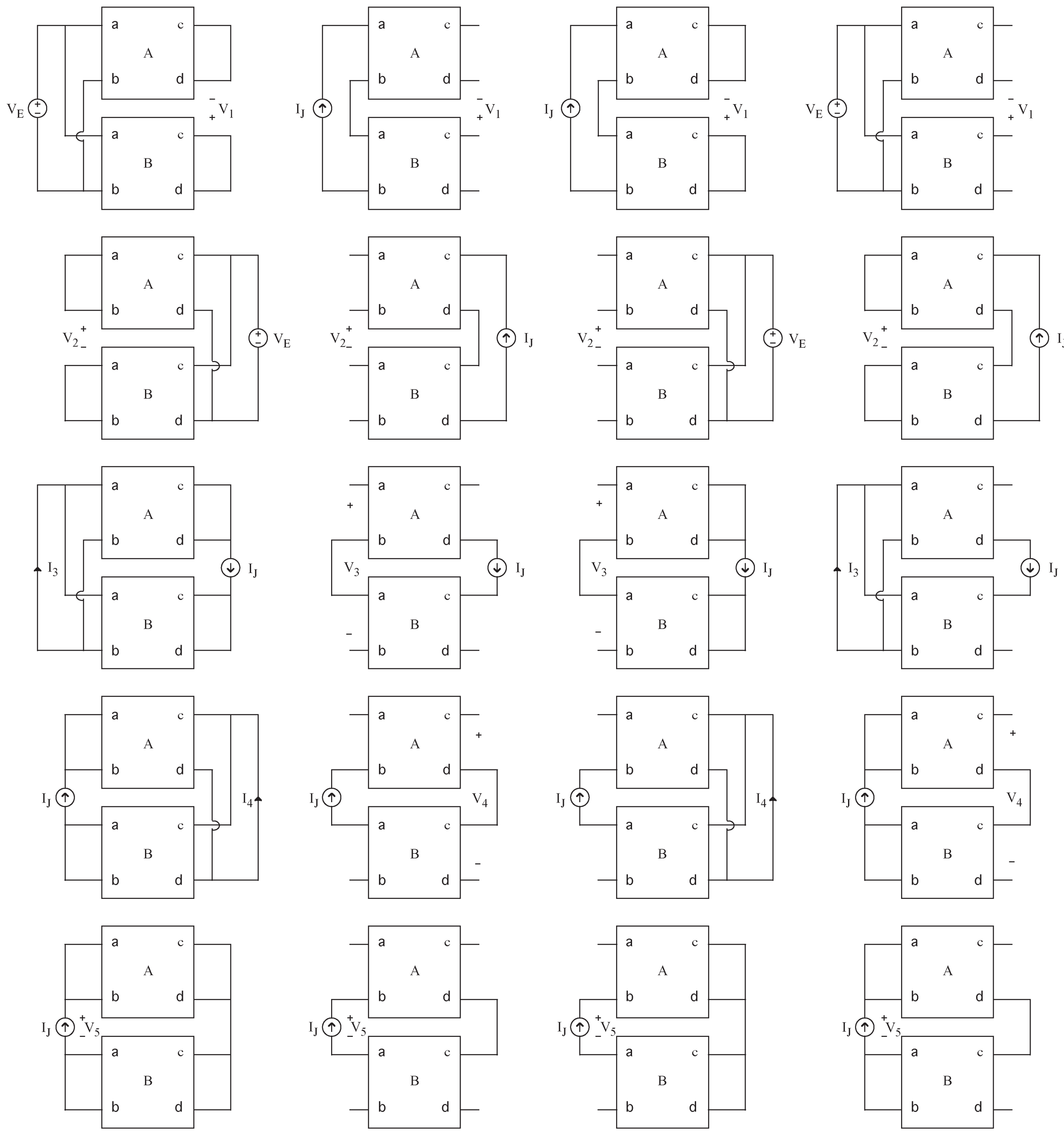

Fig. 3. Circuit sets used to ascertain whether the matrix sum rule applies or not as far as the parallel–parallel, series–series, series–parallel, and parallel–series connections are concerned.

mth circuit) are related to the input variables (V_E or I_J) in a way that condition sets (14) can be replaced by voltage/current tests. They are shown in the four columns in Fig. 3. Then, set

$$\boldsymbol{r}_A = \begin{bmatrix} r_{1A} \\ r_{2A} \end{bmatrix}' \quad \boldsymbol{r}_B = \begin{bmatrix} r_{1B} \\ r_{2B} \end{bmatrix}' \quad \boldsymbol{c}_A = \begin{bmatrix} c_{1A} \\ c_{2A} \end{bmatrix} \quad \boldsymbol{c}_B = \begin{bmatrix} c_{1B} \\ c_{2B} \end{bmatrix} \tag{17}$$

and mind that the input/output relations in the following only make sense (and thus have to be considered) for those configurations in which the involved parameters exist.

A. Test Circuits for the Parallel–Parallel Connection

By referring to the five circuits in the first column in Fig. 3, one obtains

$$V_1 = V_{bd_A} - V_{bd_B} = (r_{1A} - r_{1B})V_E \tag{18a}$$

$$V_2 = V_{bd_A} - V_{bd_B} = (r_{2A} - r_{2B})V_E \tag{18b}$$

$$I_3 = I_{ab_A} + I_{ab_B} = (c_{1A} - c_{1B})I_J \tag{18c}$$

$$I_4 = I_{cd_A} + I_{cd_B} = (c_{2A} - c_{2B})I_J \tag{18d}$$

$$V_5 = V_{bd_A} - V_{bd_B} = (k_A + k_B)I_J. \tag{18e}$$

B. Test Circuits for the Series–Series Connection

By referring to the five circuits in the second column in Fig. 3, one obtains

$$V_1 = V_{bd_A} - V_{ac_B} = (r_{1A} - r_{1B})I_J \quad (19a)$$
$$V_2 = V_{bd_A} - V_{ac_B} = (r_{2A} - r_{2B})I_J \quad (19b)$$
$$V_3 = V_{ab_A} + V_{ab_B} = (c_{1A} - c_{1B})I_J \quad (19c)$$
$$V_4 = V_{cd_A} + V_{cd_B} = (c_{2A} - c_{2B})I_J \quad (19d)$$
$$V_5 = V_{bd_A} - V_{ac_B} = (k_A + k_B)I_J. \quad (19e)$$

C. Test Circuits for the Series–Parallel Connection

By referring to the five circuits in the third column in Fig. 3, one obtains

$$V_1 = V_{bd_A} - V_{ad_B} = (r_{1A} - r_{1B})I_J \quad (20a)$$
$$V_2 = V_{bd_A} - V_{ad_B} = (r_{2A} - r_{2B})V_E \quad (20b)$$
$$V_3 = V_{ab_A} + V_{ab_B} = (c_{1A} - c_{1B})I_J \quad (20c)$$
$$I_4 = I_{cd_A} + I_{cd_B} = (c_{2A} - c_{2B})I_J \quad (20d)$$
$$V_5 = V_{bd_A} - V_{ad_B} = (k_A + k_B)I_J. \quad (20e)$$

D. Test Circuits for the Parallel–Series Connection

By referring to the five circuits in the fourth column in Fig. 3, one obtains

$$V_1 = V_{bd_A} - V_{bc_B} = (r_{1A} - r_{1B})V_E \quad (21a)$$
$$V_2 = V_{bd_A} - V_{bc_B} = (r_{2A} - r_{2B})I_J \quad (21b)$$
$$I_3 = I_{ab_A} + I_{ab_B} = (c_{1A} - c_{1B})I_J \quad (21c)$$
$$V_4 = V_{cd_A} + V_{cd_B} = (c_{2A} - c_{2B})I_J \quad (21d)$$
$$V_5 = V_{bd_A} - V_{bc_B} = (k_A + k_B)I_J. \quad (21e)$$

VI. UNIFIED VOLTAGE/CURRENT TESTS

With reference to the parallel–parallel, series–series, series–parallel, and parallel–series connections, let S_m represent the output variable of the mth test circuit, respectively, i.e.,

$$S_1 := V_1 \quad S_2 := V_2 \quad S_3 := I_3 \quad S_4 := I_4 \quad S_5 := V_5 \quad (22a)$$
$$S_1 := V_1 \quad S_2 := V_2 \quad S_3 := V_3 \quad S_4 := V_4 \quad S_5 := V_5 \quad (22b)$$
$$S_1 := V_1 \quad S_2 := V_2 \quad S_3 := V_3 \quad S_4 := I_4 \quad S_5 := V_5 \quad (22c)$$
$$S_1 := V_1 \quad S_2 := V_2 \quad S_3 := I_3 \quad S_4 := V_4 \quad S_5 := V_5. \quad (22d)$$

Then, according to (6), (18)–(21), parameter conditions (14) transform into the following tests

$$\text{DD}: \quad S_1 = S_2 = S_3 = S_4 = 0 \quad (23a)$$
$$\text{DD}: \quad S_1 = S_2 = 0 \quad S_5 \neq 0 \quad (23b)$$
$$\text{DD}: \quad S_3 = S_4 = 0 \quad S_5 \neq 0 \quad (23c)$$
$$\text{OO, OD}: \quad S_1 = S_2 = 0 \quad (23d)$$
$$\text{UU, UD}: \quad S_3 = S_4 = 0 \quad (23e)$$
$$\text{OU}: \quad \text{——}. \quad (23f)$$

Finally, from (16), (18a) and (18b), (19a) and (19b), (20a) and (20b), and (21a) and (21b), the tests for the reciprocal case are obtained

$$\text{DD, OO, OD}: \quad V_1 = V_2 = 0. \quad (24)$$

Evidently, Brune's pair of voltage tests corresponds to "DD : $V_1 = V_2 = 0$."

VII. CONCLUSION

Using a unified approach, this brief has presented both the parameter conditions and the voltage/current tests for the validity of the matrix sum rule in the four connections of two linear algebraic four-terminal components.

More than 80 years after Brune's letter, two facts have been put forward (see Section IV). First, the occurrence of a nonzero or even of an indeterminable circulating current not necessarily invalidates the matrix sum rule. Second, the balance of the "third voltages" of the two four-terminal components under individual two-port operation not necessarily makes the circulating current vanish. Anyway, these considerations should by no means detract from Brune's smart intuition.

Extensions of this approach to cover the connections of two linear algebraic n-terminal components are foreseeable.

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